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COMPARISON BETWEEN S-N-P CURVES OBTAINED FROM CONSTANT STRESS AND STEP-STRESS FATIGUE TESTS

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ABSTRACT

This Fatigue is one of the most important failure modes to be considered in many engineering applications. In actual cases, the alternate stress applied to the component may vary during its lifetime. In these situations, the direct use of S-N-P curves may be inadequate when they are based on tests where the number of cycles to failure are determined for specimens under a constant stress for a certain probability of failure.

In this paper it is shown S-N-P curves for specimens submitted each one to three different stress levels. Those curves are compared with the common S-N-P curves from constant stress tests. The Palmgren-Miner theory is applied to the obtained data from the two test methods. The results are compared and some conclusions and comments are addressed.

Keywords: Fatigue damage, cumulative damage, fatigue.

1. INTRODUCTION

Fatigue can be characterized as a phenomenon of progressive damage, being constituted basically by the nucleation and propagation of cracks, until a critical size is reached, from which the propagation happens in an unstable and uncontrollable way. Despite of the microscopic aspects have great importance for the understanding of the fatigue process, the equipment and structures designers are predominantly interested in the macroscopic aspects of the fatigue failure and in the necessary tools for its prevention during the estimated lifetime (COLLINS, 1993 and SOBCZYK & SPENCER, 1992).

For the fatigue failure prevention, one must consider the material behavior under several cyclic loads during the component lifetime. However, the fatigue characteristics of a material cannot be established from other mechanical properties. They should be measured directly, so specific mechanical tests are used to obtain the fatigue curves, denominated S-N curves. In these curves, the material life or the number of cycles to the failure, N, is expressed as function of the alternate stress range (called in this work as stress level, also) applied to the component, Sa. In general, to obtain S-N curves, a relatively high alternate stress range is applied to a specimen, kept in a constant level up to the material failure (COLLINS, 1993 and SOBCZYK & SPENCER, 1992). This procedure is repeated for several specimens, with different stress levels, progressively descending or ascending. The data are registered in a graph, with the alternate stress range (S) in the ordinate represents and the number of cycles to failure (N) in the abscissa.

However, the results of fatigue tests in specimens conducted in laboratory show significant dispersion, what even can commit the reliability of the obtained information of the curves. The random nature of fatigue would be obvious, if the structure is submitted to loads that vary randomly; but, even in laboratory conditions strictly controlled and under deterministic cyclic loads, the obtained results show considerable statistical dispersion. It occurs due to several random factors producing a huge variability in the results.

Thus, due to the inherent random nature of the fatigue data, the stochastic modeling is not only adequate but necessary. This point of view has been accepted thoroughly and, consequently, the problems associated to the fatigue stochastic theory are generally considered important and defiant in mechanics and in applied stochastic.

Due to the dispersion of the fatigue data at any level of alternate stress range, there is not only a single S-N curve for a certain material but a family of curves with the probability of failure as a related parameter. Those curves are called S-N-P curves or curves of constant probability of failure in a plot of alternate stress range versus number of cycles to failure (COLLINS, 1993). In statistics, the constant probability life is denominated percentile. The percentile 100p% of a distribution of probability F(x) is the time xp where a part p of the population will fail to this time, that is, the solution of p = F(xp).

The stochastic modeling of the fatigue is usually approached inside the reliability theory. For structures that need to operate without risk of failure, even with uncertainties, during their lifetimes, it is necessary to have an adequate probability measurement of the failure due to fatigue. For this reason, the problems of structures reliability from the point of view of fatigue are of great interest in recent research.

Furthermore, in practically all the engineering applications where fatigue is an important failure mode, the alternate stress range is not constant and changes in some way during lifetime. Such variations or load range changes, often called as load spectra, can make inadequate the direct use of the standard S-N-P curves, obtained from tests performed with constant alternate stress ranges. So, it is important to the designer to have a model, checked against experimental data, which allows him to do good estimates of the design parameters for operation under load spectra conditions. To develop this model, it is used a linear cumulative exposure method (NELSON, 1980) associated with inverse power relationship plus log-normal distribution.

This work looks for comparing S-N-P curves obtained from constant stress range fatigue tests with decreasing step-stress range fatigue tests. The comparison of the cumulative damage based on Palmgren-Miner theory is also presented.

2. S-N-P CURVES OBTAINED FROM DECREASING STEP-STRESS FATIGUE TESTS

2.1 The Experimental Data

The results of the cooled rotating bending fatigue tests performed in SAE 8620 steel specimens are presented in TABLE 1. Three alternate stress ranges were applied to each specimen, in decreasing order. In the first two steps, each specimen was submitted to 35,000 cycles of an alternate stress range of 258 MPa and to 65,000 cycles of an alternate stress range of 238 MPa. In the third step, the alternate stress ranges of 218 MPa, 198 MPa, 178 MPa and 158 MPa were applied to different specimens in a number of cycles sufficient to cause the material failure or to reach 2×10^6 cycles, characterizing the censoring type I according to NELSON, 1990. The failure was defined as the specimen full fracture. N is the number of cycles to failure or to censoring, i is the specimen number, and S is the alternate stress range or stress level.

TABLE 1: Experimental results for step-stress fatigue tests of SAE 8620 steel specimens.

i	3^{rd} step		i	3^{rd} step		i	3^{rd} step		i	3^{rd} step	
	S	N		S	N		S	N		S	N
1		212,848	12		402,834	23		900,001	34		1,000,002
2		779,027	13		992,657	24		1,309,463	35		1,802,316
3		687,887	14		948,233	25		1,587,256	36		1,589,964
4		773,889	15		833,713	26		1,624,848	37		1,445,535
5		358,453	16		815,698	27		1,534,548	38		1,721,045
6	218 MPa	708,348	17	198 MPa	866,419	28	178 MPa	1,616,273	39	158 MPa	2,000,000
7		916,372	18		1,336,343	29		1,596,508	40		2,000,000
8		952,438	19		1,118,548	30		1,606,979	41		2,000,000
9		702,855	20		871,999	31		2,000,000	42		2,000,000
10		493,165	21		909,221	32		2,000,000	43		2,000,000
11		681,238	22		825,585	33		2,000,000	44		2,000,000

2.2 Development of the Physical – Statistical Model

The model that establishes the relationship between alternate stress range and number of cycles to failure is based on two components:

(1) the deterministic one, which is the stress-time to failure relationship;

(2) the stochastic one, which is characterized by the time to failure distribution of probability.

The deterministic component of the model is the inverse power relationship (FREITAS & COLOSIMO, 1997) and, for the stochastic component, the adopted distribution to represent the time to failure data under constant alternate stress levels is the log-normal distribution PINTO (2004).

Supposing that x is the time to failure, the function of reliability of the log-normal distribution of the variable ln(x) is given by

$$R(x|S) = \Phi\left[-\frac{\ln(x) - \mu(S)}{\sigma}\right] \quad (1)$$

The shape parameter (σ) is supposed constant and the location one (μ) is function of the stress level S according to the inverse power relationship $\mu(S) = \ln\left[\left(\frac{A}{S}\right)^{\omega}\right]$.

The parameters A and ω are characteristic for the product, unit, geometry, manufacture, test method, etc.

In the case of data obtained from step-stress fatigue tests, it is adopted, for the effect of change of the stress level, the linear cumulative exposure model (NELSON, 1980). The inverse power relationship and the log-normal distributions are applied to this model. Its objective is to relate the time to failure distribution obtained with step-stress fatigue tests with the time to failure distribution under the operational cyclic loads for actual structures and components.

The cumulative exposure model supposes that:

(1) the remaining life time of a specimen only depends on the current cumulative damage fraction and on the applied alternate stress level, and is not dependent in how the damage fraction has been cumulated (Markov property (SOBCZYK & SPENCER, 1992));

(2) if the applied alternate stress level is maintained, the not failed specimens will fail according to time to failure distribution for this stress level. However, the initial point is kept at the value of the previous fraction of the cumulative failure.

Mathematically, this model is expressed below, where the time to failure distribution $F_o(x)$ under a particular configuration of the alternate step stresses applied in m levels will be obtained. Let's suppose that, for a certain configuration, the level j corresponds to the S_j level of the alternate stress range, from the instant x_{j-1} to the instant x_j, where j = 1, 2,..., m and $x_0 = 0$. The time to failure distribution at the constant level S_j of the alternate stress range is denoted by $F(S_j,x)$. The function $F_0(x)$ of the model CE is given by (NELSON, 1980):

$$F_0(x) = F(S_j, x-x_{j-1}+\xi_{j-1}),\ x_{j-1} \le x \le x_j \tag{2}$$

where ξ_{j-1} is the equivalent initial time at the level j of the alternate stress range. This time is defined as the one that would produced the same fraction of the population cumulative failure at the level j-1 of the alternate stress range. S_o ξ_{j-1} is the solution of:

$$F_0(S_{j-1}, \Delta x_{j-1} + \xi_{j-2}) = F_0(S_j, \xi_{j-1}) \tag{3}$$

where:

$$\Delta x_{j-1} = x_{j-1} - x_{j-2} \text{ and}$$
$$j = 1, 2, ..., m.$$

Substituting the expression (1) in the expression (2), the cumulative exposure model function is obtained for inverse power relationship and log-normal distribution, as

$$F_0(x) = 1 - \Phi\left\{-\frac{1}{\sigma}\ln\left[(x - x_{j-1} + \xi_{j-1})\frac{S_j^{\omega}}{A}\right]\right\},\quad x_{j-1} \le x \le x_j \tag{4}$$

The estimated parameters of the model were obtained using the maximum likelihood method (PINTO, 2004).

The TABLES 2 and 3 present the obtained estimated parameters for cumulative exposure model using the experimental data of TABLE 1.

TABLE 2: Estimated parameters of the maximum likelihood function.

Parameter	Estimator	Confidence Limit – 95%	
		Lower	Upper
A	634.82	480.31	789.33
ω	4.0977	3.2315	4.9639
σ	0.23798	0.18181	0.29415

TABLE 3: Asymptotic covariance matrix of the estimators of the maximum likelihood.

	A	ω	σ
A	6214.04	-34.7402	0.132531
ω	-34.7402	0.195293	-0.000627078
σ	0.132531	-0.000627078	0.000821348

2.3 S-N-P Curves Development

The adjusted model, that corresponds to the estimate of maximum likelihood function for the SAE 8620 steel specimens that fail after the number of cycles $x^*=10^{-4}.x$, is:

$$\hat{R}(S, x^*) = \Phi\left\{-\frac{1}{0.23798}\ln\left[x^*\left(\frac{S}{634.82}\right)^{4.0977}\right]\right\} \tag{5}$$

where Φ(.) is the cumulative distribution of a standard normal distribution (average equals 0 and standard deviation equals 1).

The expression of the adjusted model relative to the percentil 100p% is:

$$\hat{x}_p^* = \exp\left[0.23798 z_p + 4.0977(\ln 634.82 - \ln S)\right] \tag{6}$$

In expression (6), $\hat{\sigma}$ = 0.23798 is the estimate for the standard deviation of the log-normal distribution and $\hat{\omega}$ = 4.0977 and Â = 634.82 are, respectively, the estimates of the power and of the proportionality constant for the inverse power relationship. The values of these parameters are not known with so much precision as the number of significant digits seems to suggest.

Applying the adjusted model, the percentiles (x*p) confidence intervals can be obtained for the several values of the alternate stress ranges. To do that it is necessary to estimate the variance of $\hat{x}_p^*$. Using the delta method as described by FREITAS & COLOSIMO (1997), it is obtained:

$$Var\left(\hat{x}_p^*\right)=Var\left(\hat{A}\right)\left(\frac{\partial \hat{x}_p^*}{\partial A}\right)^2+Var(\hat{\omega})\left(\frac{\partial \hat{x}_p^*}{\partial \omega}\right)^2+Var(\hat{\sigma})\left(\frac{\partial \hat{x}_p^*}{\partial \sigma}\right)^2+2Cov\left(\hat{A},\hat{\omega}\right)\frac{\partial \hat{x}_p^*}{\partial A}\frac{\partial \hat{x}_p^*}{\partial \omega}+ \\ +2Cov\left(\hat{A},\hat{\sigma}\right)\frac{\partial \hat{x}_p^*}{\partial A}\frac{\partial \hat{x}_p^*}{\partial \sigma}+2Cov(\hat{\omega},\hat{\sigma})\frac{\partial \hat{x}_p^*}{\partial \omega}\frac{\partial \hat{x}_p^*}{\partial \sigma}. \tag{7}$$

The results of the percentiles confidence intervals are presented in FIGURES 1, 2 and 3.

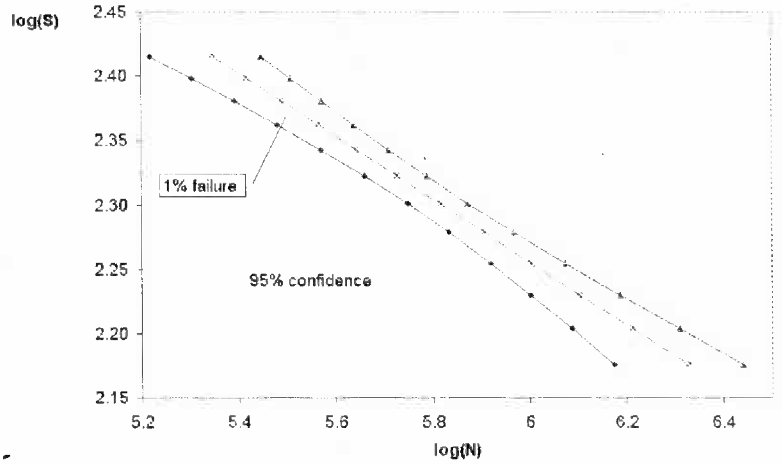

FIGURE 1: SAE 8620 steel S-N-P curve with 95% confidence limits for the percentile 1%.

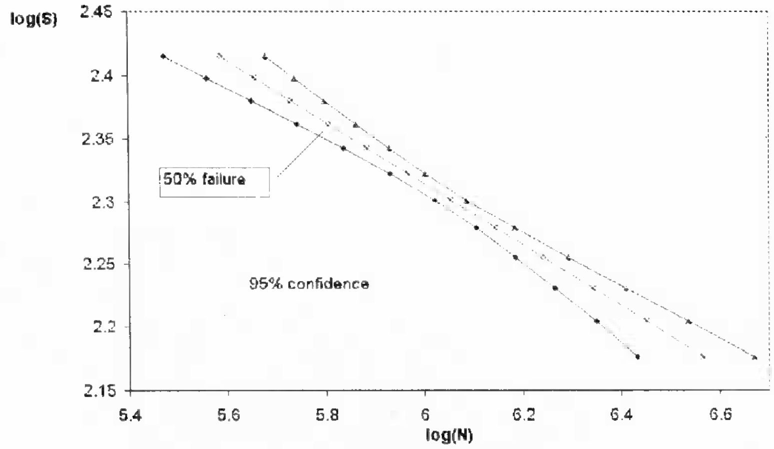

FIGURE 2: SAE 8620 steel S-N-P curve with 95% confidence limits for the percentile 50%.

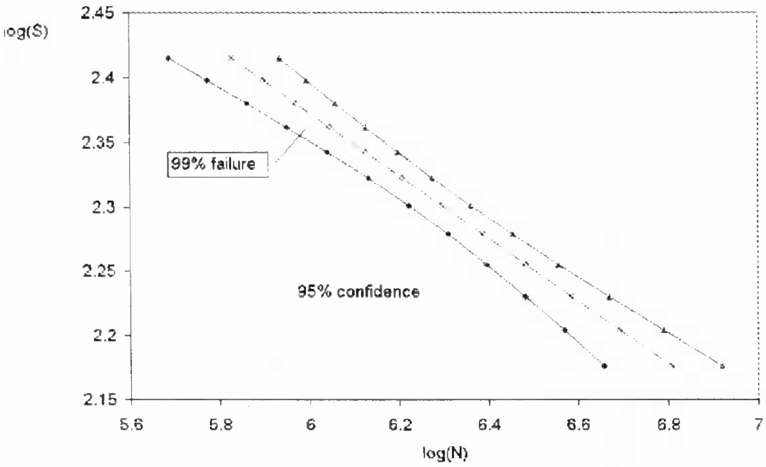

FIGURE 3: SAE 8620 steel S-N-P curve with 95% confidence limits for the percentile 99%.

The curves presented in FIGURES 1, 2 and 3 are denominated S-N-P curves. They relate the applied alternate stress level with the number of cycles to failure for a given probability of failure. As seen before, these curves were obtained from tests of specimens submitted to controlled damage (the first 2 stress steps with pre-defined number of cycles). Due to the dispersion presented by the fatigue tests each curve is presented with a range of 95% of confidence. Also, it is shown how the time to failure limits, in terms of number of cycles, change as a function of the alternate stress levels.

3. S-N-P CURVES OBTAINED FROM CONSTANT STRESS FATIGUE TESTS

3.1 The Experimental Data

The experimental data for constant stress fatigue tests are obtained from cooled rotating bending fatigue tests performed in groups of 4 or 5 specimens with 4 alternate stress levels taken between the material yielding limit and the material endurance limit.

All obtained experimental data are shown in FIGURE 4.

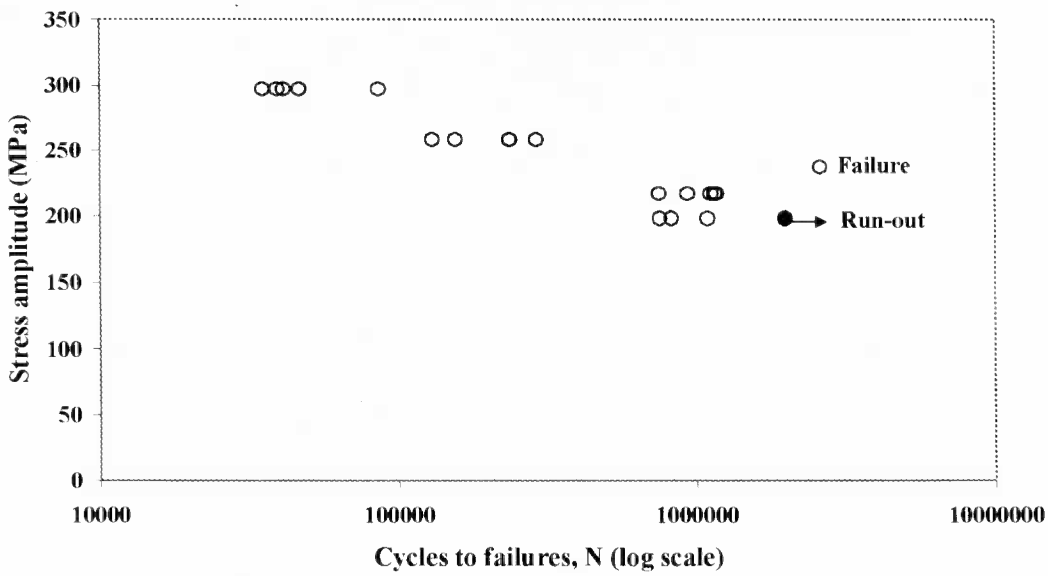

FIGURE 4: Experimental results for constant stress fatigue tests of SAE 8620 steel.

3.2 S-N-P Curves Development for Constant Alternate Stress Fatigue Tests

The S-N-P curves are obtained from a linear regression on the experimental data. The adopted equation to represent the fatigue curves is mathematically given by:

$$S = a + b \times \log[N] \quad (8)$$

where a is the linear coefficient of the straight line and b is its slope.

TABLE 4 presents the parameters of S-N-P curves obtained using the expression (8) on the experimental data of FIGURE 4.

TABLE 4: Parameters of the S-N-P curves.

Parameter	Percentiles for cooled tests		
	1%	50%	99%
a	578.9886	604.5737	630.1601
b	-65.5171	-65.5171	-65.5173

The S-N-P curves for SAE 8620 steel obtained from cooled rotating bending fatigue tests are shown in FIGURE 5. The curves are presented for three probabilities of failure, 1%, 50% and 99%. The log-normal distribution was used to obtain the curves.

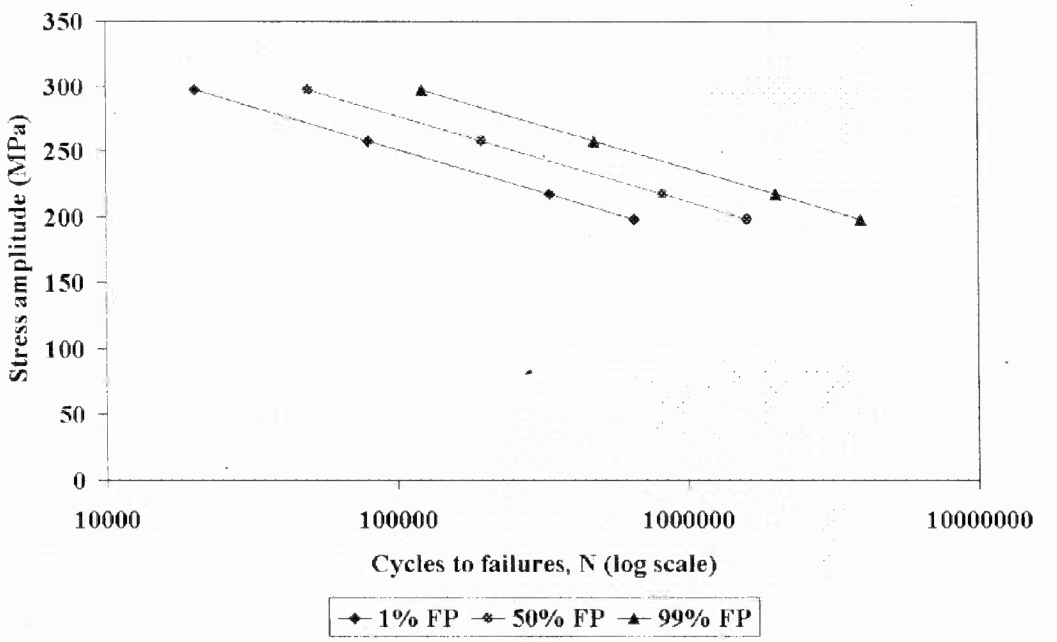

FIGURE 5: SAE 8620 steel S-N-P curves for constant stress test with probabilities of failures of 1%, 50% and 99%.

4. COMPARISON BETWEEN S-N-P CURVES OBTAINED FROM CONSTANT STRESS AND STEP-STRESS FATIGUE TESTS

Based on the developed models for alternate constant stress and step-stress fatigue tests and on the obtained results, it is shown in TABLE 5 the lifetimes as number of cycles N for 4 alternate stress ranges.

TABLE 5: SAE 8620 steel lifetimes obtained from constant stress and step-stress fatigue tests as a function of the alternate stress levels and of the probability of failure.

Alternate Stress Range (MPa)	1% PF-CS	1% 3ST	50%PF-CS	50% 3ST	99%PF-CS	99% 3ST
259	76,567	222,920	188,172	387,780	462,450	674,561
236	171,829	368,411	422,287	640,869	1,037,808	1,114,822
217	335,041	442,018	823,394	768,911	2,023,562	1,337,556
198	653,279	653,213	1,605,492	1,136,294	3,945,677	1,976,637

PF is the probability of failure CS is constant alternate stress 3ST is three steps of alternate stress

FIGURE 6 shows the plot of the TABLE 5 data of which is the comparison between SAE 8620 steel S-N-P curves obtained from constant stress and step-stress fatigue tests.

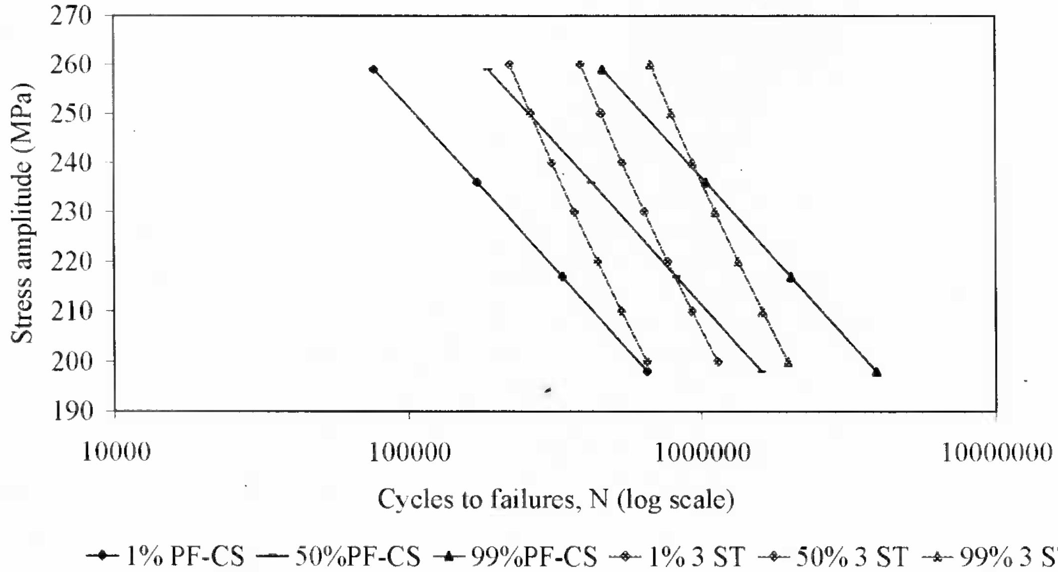

FIGURE 6: Comparison between SAE 8620 steel S-N-P curves obtained from constant stress and step-stress fatigue tests.

5. APPLICATION OF THE PALMGREN-MINER THEORY TO THE EXPERIMENTAL DATA

5.1 Introduction of damages in specimens submitted to the rotating bending fatigue

The introduction of damages in specimens submitted to the rotating bending fatigue has the purpose to assess the Palmgren-Miner damage theory.

In the TABLES 6 to 9 it is shown the number of cycles actually applied to each specimen to damage it, for each stress level.